

Highly efficient broadband conversion of light polarization by composite retarders

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Driving on an analogy with the technique of composite pulses in quantum physics, we propose highly efficient broadband polarization converters composed of sequences of ordinary retarders rotated at specific angles with respect to their fast-polarization axes. © 2012 Optical Society of America

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1. INTRODUCTION

Broadband (achromatic) polarization retarders of light have been a subject of significant interest in optics for several decades [1–3]. Such retarders are assembled by combining two or more ordinary wave plates, either of the same or different material. One of the first documented proposals was by West and Makas [4], who described achromatic combinations of plates having different dispersions of birefringence. Achromatic retarders composed of wave plates of the same material but different thicknesses were proposed by Destriau and Prouteau [5] for two birefringent plates and Pancharatnam for three plates, with which he constructed half-wave [6] and quarter-wave [7] retarders. Later Harris and co-workers proposed achromatic quarter-wave plates with six [8] and 10 identical zero-order quarter-wave plates [9].

These early studies used either the Stokes vector or the Jones vector description of light polarization. The equations of motion of these vectors, which describe the polarization change when light passes through an anisotropic optical medium [1, 2–3], are identical to basic equations in quantum mechanics: the equation for the Jones vector in a medium with zero polarization-dependent loss is identical to the Schrödinger equation [10], while the Stokes vector obeys an equation of the same form as the Bloch equation [11]. This analogy was used to link the dynamics of light polarization to the dynamics of two-state quantum systems, such as a spin-1/2 particle in a magnetic field or a two-level atom in a laser field [12,13,14,15–16]; this analogy has paved the ground for the linear optics implementation of quantum computing [17].

The composite achromatic retarders discussed above [4,5,6,7,8–9] are the precursor of the composite pulse sequences discovered later, apparently independently, in nuclear magnetic resonance (NMR) [18,19–20]. Composite pulse sequences are widely used in NMR to manipulate spins with high fidelity and robustness to parameter variations; they have a significant potential in quantum optics [21,22–23] and quantum information processing [24,25], as well. Ardavan [26] has recently recognized this analogy and proposed a broadband

composite linear retarder based on some well-known composite pulses in NMR [18,19–20].

In this paper, we use the analogy between the polarization Jones vector and the quantum state vector to propose *arbitrarily accurate* broadband polarization retarders, which promise to deliver very high polarization conversion fidelity in an arbitrarily broad range of wavelengths. There are different methods for generating arbitrary accurate composite pulses [27,28–29], which outperform the other achromatic retarders proposed earlier. However, to this end, we design novel phase-stabilized broadband (BB) composite sequences, which are more suitable for the purposes of composite retarders for practical reasons: they require far fewer wave plates to build up composite retarders of the same or comparable efficiencies. They offer very high retarding performance and, at the same time, manage to keep the number of wave plates below 10–15, thereby significantly reducing complexity and cost of fabrication.

2. BACKGROUND

Any polarization system can be viewed as composition of a retarder and a rotator [30]. A rotation at angle θ in the polarization plane is described by the Jones matrix

$$\mathcal{R}(\theta) = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}. \quad (1)$$

A retarder increases the phase of the electric field by $\varphi/2$ along the fast axis and retards it by $-\varphi/2$ along the slow axis, which can be expressed in the horizontal-vertical (HV) basis by the Jones matrix

$$\mathcal{J}(\varphi) = \begin{bmatrix} e^{i\varphi/2} & 0 \\ 0 & e^{-i\varphi/2} \end{bmatrix}, \quad (2)$$

where $\varphi = 2\pi L(n_f - n_s)/\lambda$, with λ being the vacuum wavelength, n_f and n_s the refractive indices along the fast and slow axes, respectively, and L the thickness of the plate. The most

common type of retarders are the half-wave plates ($\varphi = \pi$) and quarter-wave plates ($\varphi = \pi/2$). Since the performance of such retarders, i.e., the phase shift φ , depends strongly on the thickness and the rotary power of the plate, the traditional wave plates are not BB, as only a narrow range of wavelengths around λ acquire the desired phase shift. To this end, we use composite wave plates to create BB retarders.

Let us now consider a single polarizing birefringent plate of phase shift φ and let us introduce a system of HV polarization axes (HV basis), which are rotated by an angle θ with respect to the slow and the fast axes of the plate. The Jones matrix $\mathfrak{J}$ has the form

$$\mathfrak{J}_\theta(\varphi) = \mathfrak{R}(-\theta)\mathfrak{J}(\varphi)\mathfrak{R}(\theta). \quad (3)$$

In the left-right circular polarization (LR) basis, this matrix obtains the form $\mathbf{J}_\theta(\varphi) = \mathbf{W}^{-1}\mathfrak{J}_\theta(\varphi)\mathbf{W}$, where $\mathbf{W}$ connects the HV and LR polarization bases:

$$\mathbf{W} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ -i & i \end{bmatrix}. \quad (4)$$

Explicitly, the Jones matrix in the LR basis is

$$\mathbf{J}_\theta(\varphi) = \begin{bmatrix} \cos(\varphi/2) & i \sin(\varphi/2)e^{2i\theta} \\ i \sin(\varphi/2)e^{-2i\theta} & \cos(\varphi/2) \end{bmatrix}. \quad (5)$$

Half- and quarter-wave plates rotated by angle θ , $(\lambda/2)_\theta$ and $(\lambda/4)_\theta$, are respectively described by $\mathbf{J}_\theta(\pi)$ and $\mathbf{J}_\theta(\pi/2)$.

3. COMPOSITE POLARIZATION RETARDERS

Our objective is to construct retarders that are robust to variations in the phase shift φ at selected value(s) of this shift. Such retarders tolerate imperfect rotary power φ/L and plate thickness L and also operate over a broad range of wavelengths λ . To this end, we replace the single retarder with a sequence of N retarders, each with phase shift φ_k and rotated by angle θ_k , described in the LR basis by the Jones matrix (read from right to left):

$$\mathbf{J}^{(N)} = \mathbf{J}_{\theta_N}(\varphi_N)\mathbf{J}_{\theta_{N-1}}(\varphi_{N-1}) \cdots \mathbf{J}_{\theta_1}(\varphi_1). \quad (6)$$

The efficiency of the composite retarder is measured by the fidelity $F = \frac{1}{2}\text{Tr}(\mathbf{J}_0^{-1}\mathbf{J}^{(N)})$ [26], where $\mathbf{J}^{(N)}$ is the achieved and $\mathbf{J}_0$ is the target Jones matrix.

First, we show how to construct a BB *half-wave retarder*, which inverts light polarization with high fidelity in a broad range of phase shifts φ around π . Its Jones matrix in the LR basis reads (up to a global phase factor):

$$\mathbf{J}_0(\pi) = \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}. \quad (7)$$

We compose a sequence of an odd number N of $\lambda/2$ plates ($\varphi_k = \varphi = \pi$), each rotated by θ_k : $(\lambda/2)_{\theta_N}(\lambda/2)_{\theta_{N-1}} \cdots (\lambda/2)_{\theta_1}$ [read from right to left; see Fig. 1(a)]. The phase θ_1 does not change the performance of our retarders. Therefore, for convenience, we set $\theta_1 = 0$ and we are left with $N - 1$ relative rotation angles, which are treated as free parameters.

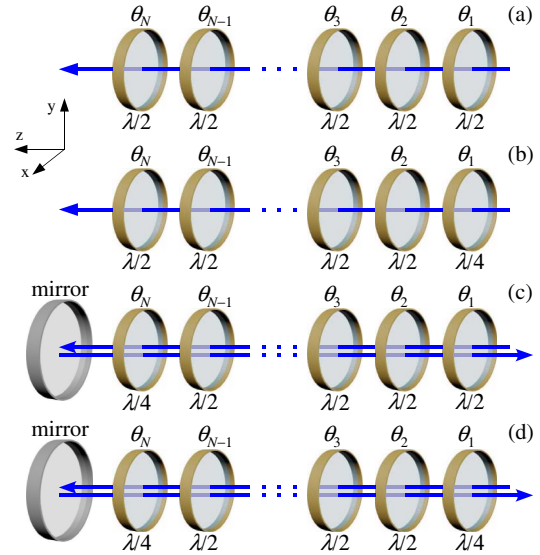


Fig. 1. (Color online) Implementations of different BB composite retarders composed of N half- and quarter-wave plates, each rotated by angle θ_k ($k = 1, 2, \dots, N$) with respect to the fast axis. (a) BB half-wave retarder composed of N half-wave plates; (b) BB quarter-wave retarder composed of $N - 1$ half-wave plates and a quarter-wave plate; (c) BB half-wave retarder composed of $N - 1$ half-wave plates, a quarter-wave plate, and a mirror; (d) BB quarter-wave retarder composed of $N - 2$ half-wave plates, two quarter-wave plates, and a mirror.

Next, we obtain the composite Jones matrix (6)

$$\mathbf{J}^{(N)} = \mathbf{J}_{\theta_N}(\pi)\mathbf{J}_{\theta_{N-1}}(\pi) \cdots \mathbf{J}_{\theta_1}(\pi), \quad (8)$$

and set $\mathbf{J}^{(N)} = \mathbf{J}_0(\pi)$ at $\varphi = \pi$, which leaves us with $N - 2$ independent angles θ_k to vary. We then nullify as many lowest order derivatives of $\mathbf{J}_{12}^{(N)}$ versus the phase shift φ at $\varphi = \pi$ as possible. We thus obtain a system of nonlinear algebraic equations for the $N - 2$ rotation angles θ_k . For our composite retarder composed of $\lambda/2$ plates only, all odd-order derivatives vanish at $\varphi = \pi$; hence, $N - 2$ rotation angles allow us to nullify the first $N - 2$ complex derivatives:

$$[\partial_\varphi^k \mathbf{J}_{12}^{(N)}]_{\varphi=\pi} = 0 \quad (k = 1, 2, \dots, N - 2), \quad (9)$$

as well as the real or imaginary part of the next nonzero derivative (of order $N - 1$):

$$\text{Re}[\partial_\varphi^{N-1} \mathbf{J}_{12}^{(N)}]_{\varphi=\pi} = 0 \quad \text{or} \quad \text{Im}[\partial_\varphi^{N-1} \mathbf{J}_{12}^{(N)}]_{\varphi=\pi} = 0. \quad (10)$$

Solutions to Eqs. (9) and (10) provide BB half-wave retarders. Longer retarders, of larger number N of constituent wave plates, provide higher order of stability against variations of the phase shift φ and the light wavelength λ . Examples of BB half-wave retarders are listed in Table 1. Their fidelities and phase retardances are illustrated, respectively, in Fig. 2(a) and Fig. 3(a).

We can construct in the same manner various BB *quarter-wave retarders*. Their Jones matrix in the LR basis is

$$\mathbf{J}_0(\pm\pi/2) = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & \pm i \\ \pm i & 1 \end{bmatrix}. \quad (11)$$

Table 1. Rotation Angles θ_k (in Degrees) for BB Retarders with Different Number N of Constituent Half-Wave Plates^a

N	Rotation Angles ($\theta_1; \theta_2; \dots; \theta_{N-1}; \theta_N$)
(a) Half-wave retarders	
5	(0; 52.2; 336.7; 336.7; 52.2)
9	(0; 158; 59.9; 45.5; 151.6; 174.4; 108.4; 0.5; 58.9)
13	(0; 109.1; 175; 49; 91.6; 9.7; 172.8; 28.6; 127.3; 131.2; 174.8; 76.1; 22.1)
(b) Quarter-wave retarders	
4	(0; 69.3; 318.6; 69.3)
5	(0; 48.6; 325.8; 325.8; 48.6)
6	(0; 116.6; 69.1; 175; 69.1; 116.6)
8	(0; 104.8; 103.6; 32.5; 149.6; 52.6; 74.3; 137.6)
10	(0; 34.3; 97.1; 120.6; 142.7; 50.7; 8.9; 121.2; 64.9; 77)
(c) Half-wave retarders with a mirror	
2	(30; 150)
3	(39; 10.3; 122.7)
4	(22; 73.4; 171.6; 150.2)
5	(9.4; 115.1; 154.7; 51; 3.8)
10	(110.3; 161.4; 18.8; 111.7; 96; 84.9; 136.2; 65.7; 5.9; 67.1)
(d) Quarter-wave retarders with a mirror	
4	(45; 19.3; 113.6; 166)
5	(45; 131.1; 169.3; 39.7; 115.5)
6	(45; 130.3; 164.6; 25.1; 83; 162)
8	(45; 141.5; 104.9; 127.2; 43.2; 65.9; 6.2; 112)
9	(45; 152.7; 73.2; 30.1; 1.2; 144.4; 94.4; 29; 127.4)
10	(45; 126.6; 165.5; 145.1; 154.9; 64; 43; 25.3; 87.1; 156.5)

^a(a) Half-wave BB composite retarders; see Fig. 1(a). (b) Quarter-wave BB composite retarders; see Fig. 1(b). (c) Half-wave BB composite retarders with a mirror; see Fig. 1(c). (d) Quarter-wave BB composite retarders with a mirror; see Fig. 1(d).

We have found that the most suitable composite sequence consists of $N - 1$ $\lambda/2$ plates ($\varphi = \pi$) and a $\lambda/4$ plate ($\varphi = \pi/2$): $(\lambda/2)_{\theta_N}(\lambda/2)_{\theta_{N-1}} \dots (\lambda/2)_{\theta_2}(\lambda/4)_0$ [read from right to left; see Fig. 1(b)]. The corresponding Jones matrix is

$$\mathbf{J}^{(N)} = \mathbf{J}_{\theta_N}(\pi) \mathbf{J}_{\theta_{N-1}}(\pi) \dots \mathbf{J}_{\theta_2}(\pi) \mathbf{J}_0(\pi/2). \quad (12)$$

There are $N - 1$ free phases θ_k , with which we can nullify the first $\lfloor (N - 1)/2 \rfloor$ complex derivatives

$$[\partial_{\varphi}^k \mathbf{J}_{12}^{(N)}]_{\varphi=\pi} = 0 \quad (k = 1, 2, \dots, \lfloor (N - 1)/2 \rfloor), \quad (13)$$

where $\lfloor x \rfloor$ denotes the integer part of x . For even N , we can nullify also the real or imaginary part of the next nonzero derivative (of order $N/2$):

$$\text{Re}[\partial_{\varphi}^{N/2} \mathbf{J}_{12}^{(N)}]_{\varphi=\pi} = 0 \quad \text{or} \quad \text{Im}[\partial_{\varphi}^{N/2} \mathbf{J}_{12}^{(N)}]_{\varphi=\pi} = 0. \quad (14)$$

Table 1 lists a set of phases that produce BB quarter-wave retarders; fidelities and phase retardances are shown, respectively, in Figs. 2(b) and 3(b).

4. COMPOSITE POLARIZATION RETARDERS WITH A MIRROR

We show now how to construct BB quarter-wave and half-wave retarders by placing a mirror at the end of the composite

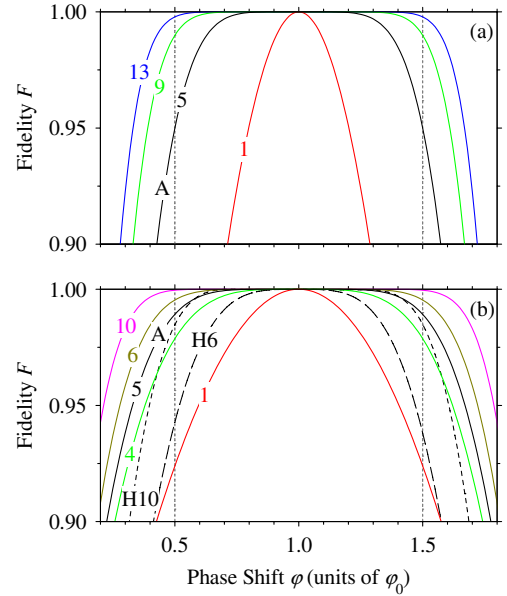


Fig. 2. (Color online) Fidelity F versus phase shift φ for BB half-wave retarders [frame (a) with $\varphi_0 = \pi$; see Fig. 1(a)] and BB quarter-wave retarders [frame (b) with $\varphi_0 = \pi/2$; see Fig. 1(b)], for different number of constituent plates N . The rotation angles are given in Table 1. The fidelities of the single-plate retarder and the BB1 retarder [26] are shown with labels “1” and “A,” while the fidelities of the six- and 10-plate retarders of Harris and co-workers [8,9] are shown by dashed lines with labels “H6” and “H10.” As one can clearly see, our retarders outperform the others for $N > 5$.

sequence of plates. Thus the retarders are effectively twice as long and we can reduce the number of wave plates while maintaining the fidelity. We note that, if the incident light “sees” a wave plate rotated at an angle θ , then the reflected light will “see” the same wave plate rotated at the angle $-\theta$.

For the BB half-wave retarder, the most efficient sequence is composed of $N - 1$ half-wave plates ($\varphi = \pi$) and a

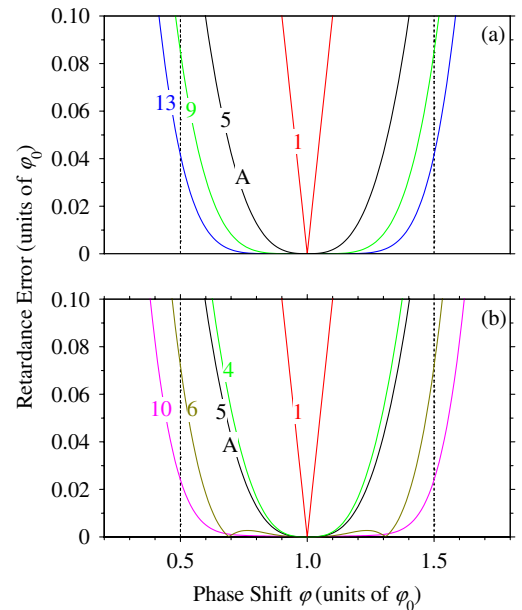


Fig. 3. (Color online) Retardance error versus phase shift φ for BB half-wave retarders without a mirror [frame (a) with $\varphi_0 = \pi$; see Fig. 1(a)] and BB quarter-wave retarders without a mirror [frame (b) with $\varphi_0 = \pi/2$; see Fig. 1(b)], for different number of constituent plates N . The rotation angles are given in Table 1.

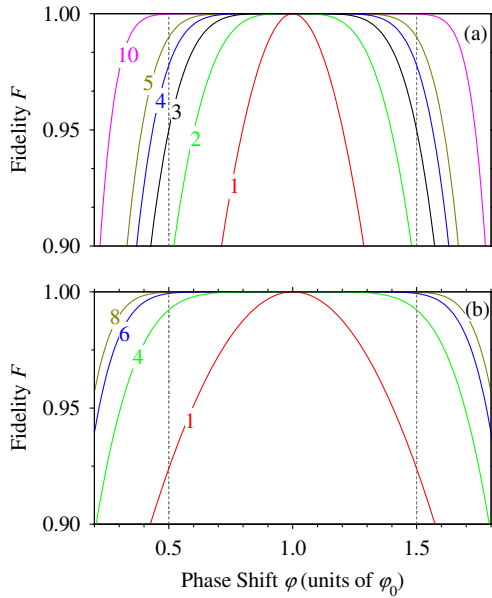


Fig. 4. (Color online) Fidelity F versus phase shift φ for BB half-wave retarders with a mirror [frame (a) with $\varphi_0 = \pi$; see Fig. 1(c)] and BB quarter-wave retarders with a mirror [frame (b) with $\varphi_0 = \pi/2$; see Fig. 1(d)], for different number of constituent plates N . The rotation angles are given in Table 1.

quarter-wave plate ($\varphi = \pi/2$): $M(\lambda/4)_{\theta_N}(\lambda/2)_{\theta_{N-1}} \cdots (\lambda/2)_{\theta_2}(\lambda/2)_{\theta_1}$ [read from right to left; see Fig. 1(c)]. The total Jones matrix is

$$\mathbf{J}^{(2N)} = \mathbf{J}_{-\theta_1}(\pi)\mathbf{J}_{-\theta_2}(\pi) \cdots \mathbf{J}_{-\theta_{N-1}}(\pi)\mathbf{J}_{-\theta_N}(\pi/2)\sigma_x \times \mathbf{J}_{\theta_N}(\pi/2)\mathbf{J}_{\theta_{N-1}}(\pi) \cdots \mathbf{J}_{\theta_2}(\pi)\mathbf{J}_{\theta_1}(\pi), \quad (15)$$

where the Pauli matrix σ_x describes the mirror in the LR basis.

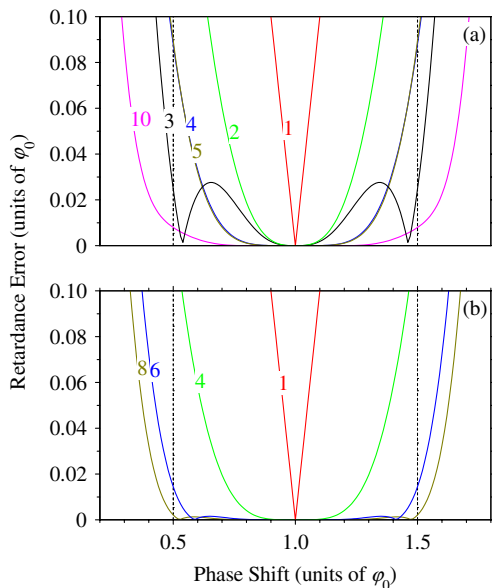


Fig. 5. (Color online) Retardance error versus phase shift φ for BB half-wave retarders with a mirror [frame (a) with $\varphi_0 = \pi$; see Fig. 1(c)] and BB quarter-wave retarders with a mirror [frame (b) with $\varphi_0 = \pi/2$; see Fig. 1(d)], for different number of constituent plates N . The rotation angles are given in Table 1.

First we impose $\mathbf{J}^{(2N)} = \mathbf{J}_0(\pi)$. We are left with $N - 1$ rotation angles θ_k , which we use to nullify the first $N - 2$ (complex) derivatives

$$[\partial_\varphi^k \mathbf{J}_{12}^{(2N)}]_{\varphi=\pi} = 0 \quad (k = 1, 2, \dots, N - 2), \quad (16)$$

and the real or the imaginary part of the next nonzero derivative (of order $N - 1$):

$$\text{Re}[\partial_\varphi^{N-1} \mathbf{J}_{12}^{(2N)}]_{\varphi=\pi} = 0 \quad \text{or} \quad \text{Im}[\partial_\varphi^{N-1} \mathbf{J}_{12}^{(2N)}]_{\varphi=\pi} = 0. \quad (17)$$

Exemplary solutions are listed in Table 1; fidelities and phase retardances, respectively, are shown in Figs. 4(a) and 5(a).

For the BB quarter-wave retarder, the most suitable sequence is composed of $N - 2$ half-wave plates ($\varphi = \pi$) surrounded by two quarter-wave plates ($\varphi = \pi/2$) as follows: $M(\lambda/4)_{\theta_N}(\lambda/2)_{\theta_{N-1}} \cdots (\lambda/2)_{\theta_2}(\lambda/4)_{\theta_1}$ [read from right to left; see Fig. 1(d)]. The total Jones matrix is

$$\mathbf{J}^{(2N)} = \mathbf{J}_{-\theta_1}(\pi/2)\mathbf{J}_{-\theta_2}(\pi) \cdots \mathbf{J}_{-\theta_{N-1}}(\pi)\mathbf{J}_{-\theta_N}(\pi/2)\sigma_x \times \mathbf{J}_{\theta_N}(\pi/2)\mathbf{J}_{\theta_{N-1}}(\pi) \cdots \mathbf{J}_{\theta_2}(\pi)\mathbf{J}_{\theta_1}(\pi/2). \quad (18)$$

We impose $\mathbf{J}^{(2N)} = \mathbf{J}_0(\pi/2)$, at the expense of two rotating angles θ_k , and set the first $N - 2$ (complex) derivatives to zero:

$$[\partial_\varphi^k \mathbf{J}_{12}^{(2N)}]_{\varphi=\pi} = 0 \quad (k = 1, 2, \dots, N - 2). \quad (19)$$

Exemplary solutions are listed in Table 1; fidelities and phase retardances are shown, respectively, in Figs. 4(b) and 5(b).

5. CONCLUSION

The presented composite retarders are advantageous, as they allow us to manipulate polarization only by varying the rotation angle of the single wave plates. Therefore, they offer robustness against variations of the parameters of both the crystal and the light field. These include the crystal temperature, the wavelength of the electric field, the crystal length, and the angle of incidence. In addition, the proposed retarders significantly outperform the existing BB retarders [8,26] for more than five individual plates. An experimental implementation with standard retarders available in most laboratories should be straightforward.

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